

## ENHANCED RAY THEORY

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Received 1 January 2000

Revised 1 June 2000

The numerical problems of SWAM'99 workshop are quite challenging for any method of sound field calculation. This report presents a detailed description of the *enhanced ray theory* approach briefly outlined in Ref. 2. It contains a new method of phase and amplitude computation along the ray, a new method of calculation of eigenrays, and a new method of analytic approximation of sound-speed and density. An application of these methods is presented.

### 1. Caustics

One of the intrinsic flaws of classical ray theory<sup>1</sup> is an infinite field at caustics. The caustic problem was treated in many papers (see Refs. 3–5). All these investigations, without exception, require knowledge of the *global* distribution of ray trajectories in the vicinity of caustics. This feature gives rise to substantial difficulties in the numerical implementation. Here, we derive *local* representations of ray trajectories giving finite values at caustics.

The Fourier transform  $U(\mathbf{r}, \omega)$  of the acoustic pressure  $P(\mathbf{r}, t)$  satisfies the Helmholtz equation in an inhomogeneous acoustic medium

$$\nabla^2 U(\mathbf{r}) - (\nabla U(\mathbf{r}), \nabla \mu(\mathbf{r})) + k^2 n^2(\mathbf{r}) U(\mathbf{r}) = \frac{\delta(\mathbf{r} - \mathbf{r}_0)}{4\pi}. \quad (1.1)$$

where  $(\nabla U(\mathbf{r}), \nabla \mu(\mathbf{r}))$  signifies the dot product of two gradients. Here  $\mu(\mathbf{r}) = \ln(\rho(\mathbf{r})/\rho_0)$ ,  $k = \omega/c_0$  and  $n(\mathbf{r}) = c_0/c(\mathbf{r})$ , where  $\rho_0$  is the mass density,  $c_0$  is the sound-speed, and  $\omega$  is the angular frequency. Consider the following solution of Eq. (1.1):

$$U(\mathbf{r}) = \exp(ik\tau(\mathbf{r})), \quad k = \frac{\omega}{c_0}, \quad (1.2)$$

where the function  $\tau(\mathbf{r})$  satisfies

$$(\nabla \tau, \nabla \tau) + \frac{i}{k} ((\nabla \tau, \nabla \mu) - (\nabla, \nabla \tau)) - n^2(\mathbf{r}) = 0. \quad (1.3)$$

For  $k \rightarrow \infty$  the equation governing  $\tau(\mathbf{r})$  is a first-order partial differential equation which can be solved by the method of characteristics rendering the rays  $\mathbf{R}(s, \theta_0, \phi_0)$  which are 3-D curves characterized by the arclength  $s$  along the ray and the two launch angles  $\theta_0, \phi_0$ . When

$$\nabla \tau = q(s) \mathbf{t}(s, \theta_0, \phi_0), \quad |\mathbf{t}| = 1, \quad (1.4)$$

